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Optimal management of uneven-aged Norway spruce stands

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ABSTRACT

An optimization model is specified to analyze forest management without any restrictions on the forest management system. The data on forest growth comes from unique field experiments and is used to estimate a nonlinear transition matrix or size-structured model for Norway spruce. The objective function includes detailed harvesting cost specifications and the optimization problem is solved in its most general dynamic form. In optimal uneven-aged management, stand density is shown to be dominated by limitations in natural regeneration. If the goal is volume maximization, even-aged management with artificial regeneration (and thinnings from above) is superior to uneven-aged management. After including regeneration and harvesting costs, the interest rate, and the price differential between saw timber and pulpwood, uneven-aged management becomes superior to even-aged management. However, in the short term the superiority is conditional on the initial stand state.

1. Introduction

Studies on optimal forest management rest strongly on the classical rotation framework. This approach has proven to be theoretically valid and open to numerous extensions. However, at a more general level it can be asked whether the underlying forest management specification, i.e. producing trees in even-aged cohorts, is superior to all possible alternatives. This study estimates a size-structured stand growth model, and applies the model in analyzing optimal timber production without any preconditions on the forest management system. Such a model will yield new understanding on the profitability of forest management alternatives and several surprising results for Norway spruce, a tree species that in Finland and Sweden has been managed – almost without exception – as even-aged stands.

The seminal paper on optimal uneven-aged forestry is by Adams and Ek (1974). They specify the problem in a general dynamic setup but apply, as most of their followers, several simplifications in searching the optimal solutions. Typically studies compute static steady states and maximize physical yield or stumpage revenues. More general specifications are used by Haight (1985, 1987), Getz and Haight (1989), Haight and Monserud (1990), Buongiorno et al. (1995) who have applied both transition matrix and single-tree growth models together with dynamic optimization methods.

During the last 60 years or so conifer forests in Finland and Sweden have been managed under a planting/thinning-from-below/clearcut management system (Siiskonen, 2007). This can be criticized by referring to several optimization studies, which almost without exception, have shown that it is optimal to thin coniferous stands from above (Haight and Monserud, 1990; Valsta, 1992; Pukkala and Miina, 1998; Hyytiäinen et al., 2004, 2010). Although this result is a rather serious anomaly to the orthodox thinning-from-below/clearcut management system, Nordic studies open to a thinning-from-above/no clearcuts alternative have been rare and almost nonexistent. A small set of Nordic studies has performed field experiments where a number of similar sites have been managed as even- or uneven-aged systems over several decades. Andreassen and Øyen (2002) compare clearcutting, tree selection, and group selection in six initially mature Norway spruce stands. Clearcutting is found to yield slightly higher net present value compared to uneven-aged management but all three forest management methods were found to be economically reasonable. Lundqvist et al. (2007) found volume increments to be higher in plots thinned from above than from plots thinned from below and that natural regeneration was sufficient for replacing the trees harvested.

Tahvonen (2007, 2009) applies a transition matrix model from Kolström (1993) and Pukkala and Kolström (1988). According to the results the optimal solution converges to a cyclical harvesting strategy with uneven-aged stand structure. A rough comparison of

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this solution with an optimal even-aged solution (no harvesting, no regeneration costs) yields the result that the uneven-aged solution appears to be superior. One element with high uncertainty in this comparison was the density free natural regeneration specification in Kolström (1993).

Pukkala et al. (2009) estimated individual tree diameter increment, height, survival, and ingrowth models for major Finnish tree species. They did not perform economic analysis but found that sustainable harvesting may yield 5–7 m^3 ha^{-1} a^{-1}—output that compares well with the output under even-aged management (see also Bollandsås et al., 2008).

Pukkala et al. (2010) used their estimated model in Pukkala et al. (2009) and computed static investment-efficient steady states following the ideas by Duerr and Bond (1952) and Chang (1981). Excluding some most fertile southern Finland site types and low rates of interest, uneven-aged forestry was found superior to even-aged forestry both for Scots pine and Norway spruce.

These new studies are in interesting contrast with the existing forest policy and practices in Finland and Sweden. However, the understanding on uneven-aged management of Norway spruce is still restrictive and far from complete. The existing studies specify the management of uneven-aged forestry *ad hoc*, apply strongly simplified growth models or are based on entirely static optimization framework. This study aims to go beyond these limitations by using data from unique uneven-aged field experiments to estimate a transition matrix and ingrowth models (cf. Liang et al., 2005) that are suitable for computing dynamically optimal forest management solutions. In addition, this study applies detailed harvesting cost models for logging conditions in uneven-aged forestry. These extensions enable to perform theoretically sound and general comparisons of the relative profitability of even- and uneven-aged management without restricting the analysis to steady states.

2. Material and methods

2.1. The optimization problem

This study applies the size-structured transition matrix (Usher, 1966, 1969) model for describing the stand growth. Let x_{st}, $s=1,\ldots,n$, $t=0,1,\ldots$ denote the number of trees in size class s at period t. Let $\alpha_s(\boldsymbol{x}_t)$, $s=1,\ldots,n$, $t=0,1,\ldots$ denote the fraction of trees that reaches the next size class for period $t+1$. This fraction is size class specific and depends on the stand density. Denote the fraction of trees that die by $\mu_s(\mathbf{x}_t)$, $s=1,\ldots,n$, $t=0,1,\ldots$. The remaining fraction, i.e. $1-\alpha_s(x_t)-\mu_s(x_t)(\leq 1)$ will stay in their present size class. Ingrowth, or the number of trees that enters the smallest size class, is given by $\phi(\mathbf{x}_t)$ and the number of trees cut from each size class by h_{st}, $s=1,\ldots,n$, $t=0,1,\ldots$. The stand development can now be specified as the set of the following difference equations:

$$x_{1,t+1}=\phi(\boldsymbol{x}_t)+[1-\alpha_1(\boldsymbol{x}_t)-\mu_1(\boldsymbol{x}_t)]x_{1t}-h_{1t}, \tag{1}$$

$$x_{s+1,t+1}=\alpha_s(\boldsymbol{x}_t)x_{st}+[1-\alpha_{s+1}(\boldsymbol{x}_t)-\mu_{s+1}(\boldsymbol{x}_t)]x_{s+1,t}-h_{s+1,t},\quad s=1,\ldots,n-2 \tag{2}$$

$$x_{n,t+1}=\alpha_{n-1}(\boldsymbol{x}_t)x_{n-1,t}+[1-\mu_n(\boldsymbol{x}_t)]x_{nt}-h_{nt}. \tag{3}$$

Note that the state of the forest, i.e. the number of trees, is given at the beginning of each period and the number of trees harvested at the end of each period. According to Eq. (3) trees that have reached the largest size class n remain in this size class until they die or are harvested.

Let the gross revenues for a harvest be denoted as R_t. The revenue is given as

$$R_t=\sum_{s=1}^{n}h_{st}(\omega_{s1}p_1+\omega_{s2}p_2) \tag{4}$$

where $\omega_{si},\ s=1,\ldots,n,\ i=1,2$ denote the saw logs and pulpwood volume of a trunk in m^3 and p_i, $i=1,2$ denote the corresponding roadside prices (in €). Logging and hauling costs are denoted by C_t and depend on the volume of timber harvested and on the dimensions of harvested trees:

$$C_t=C(\boldsymbol{h}_t,\ \boldsymbol{d})+C_f, \tag{5}$$

where C_f denotes the fixed harvesting cost and vector $\boldsymbol{d}=d_1\ldots,d_n$ denotes the tree diameter at breast height in cm. Given that $b=1/(1+r)$ denotes the discount factor and r the interest rate, the problem of maximizing net revenues from a given stand can be given as:

$$\max_{\{h_{st},s=1,\ldots,n,t=0,1,\ldots\}} V_1(\boldsymbol{x}_0)=\sum_{t=0}^{\infty}(R_t-C_t)b^t, \tag{6}$$

subject to Eqs. (1)–(5) and subject to the following non-negativity constraints and stand initial size distribution:

$$h_{st}\geq 0, x_{st}\geq 0, s=1,\ldots,n, t=0,1,\ldots,\quad x_{s0}, s=1,\ldots,n \text{ given.} \tag{7a,b}$$

The problem is complicated by the presence of fixed harvesting costs because it may not be optimal to cut the forest every (3-year) time period. A simple, albeit somewhat unsatisfactory, possibility that takes this into account is to allow cutting every $k\geq 1$ periods only. By varying k and observing the value of the objective function it is then possible to find the optimal cutting period. Formally this feature can be specified by adding a restriction:

$$h_{st}=0 \quad \text{when}\, t\neq k, 2k, 3k, \ldots, \tag{8}$$

where the value of k is a positive integer.

2.2. Model estimation and data

Data from two long-term experiments (Vessari and Honka) of uneven-aged forest management were used to develop models for the probability that a tree moves to the next diameter class (upgrowth), and the probability that the tree dies during the next 3-year period. Another model was fitted for ingrowth, expressed as the number of trees that will pass the 5 cm diameter limit during the following 3-year period. Three years was selected as the time step because the plots of the experiments had been measured at 3-year intervals. As in Pukkala et al. (2009), 5 cm was taken as the lower limit of the smallest diameter class, and 4 cm was used as the class width. The midpoints of diameter classes were therefore 7, 11, 15,. . .,43 cm.

The two experiments have 92 sample plots in total. The plots have been measured seven times, starting in 1990. This results in six 3-year measurement intervals. The plots are located in Central Finland, and they represent *Oxalis-Myrtillus* (OMT) and *Myrtillus* (MT) forest site types, which are typical sites for Norway spruce (see Cajander, 1949 for forest site types). The annual temperature sum in the study sites is approximately 1200 (°Cd) For ingrowth estimation, the total number of observations was 459 (plots with two measurements at 3-year intervals). The models for the probability of upgrowth were based on 19811 observations, of which 17,758 were trees that remained in the same diameter class and 2,058 were trees that moved up one class. For mortality modeling, the observations were the same 19811 trees that survived for the

Table 1
Sawn timber and pulpwood volumes m^3 per size classes.

Diameter (cm)	7	11	15	19	23	27	31	35	39	43
Sawn timber	0	0	0	0.14136	0.29572	0.45456	0.66913	0.88761	1.12891	1.39180
Pulp wood	0.01189	0.05138	0.12136	0.08262	0.06083	0.06703	0.04773	0.04596	0.04672	0.04119

3-year period, plus 971 additional trees that died between the two measurements.

Ingrowth for the 5–9 cm diameter class is given by

$$x_{1,t+1} = e^{2.1368+0.104\sqrt{N_t}-0.107y_t} - 1, \tag{9}$$

where N_t is the total number of trees and y_t is the stand total basal area (in m^2). Thus, ingrowth increases with the number of trees and decreases with the total basal area. The degree of explained variance (R^2) of the ingrowth model is 0.391.

Let variable $y_{st}, s = 1,\ldots,n, t = 0,1,\ldots$ denote the sum of basal areas of trees in size classes larger than size class s including half of the basal area of trees in size class s. Thus

$$y_{st} = \frac{x_{st}\pi(d_s/2)^2 10^{-4}}{2} + \sum_{k=s+1}^{n} x_{st}\pi\left(\frac{d_s}{2}\right)^2 10^{-4}, \quad s = 1, \ldots, n. \tag{10}$$

The fraction of trees that move to the next size class during the next 3 years equals:

$$\alpha_s = \left\{1 + e^{[3.752-2.560\sqrt{d_s}+0.296d_s+0.849\ \ln(y_t)+0.035y_{st}]}\right\}^{-1}. \tag{11}$$

Thus, this fraction decreases with the total basal area and the basal area of larger diameter trees. In addition, it increases with low diameter levels but decreases when the diameter reaches a level of 18 cm. The percentage of correct predictions of the upgrowth model was 89.6%, and the Nagelkerke R^2 was 0.173.

A fraction of trees that die is given by

$$\mu_s = \left\{1 + e^{[3.606-0.075y_{st}+0.997\ \ln(d_s)]}\right\}^{-1}. \tag{12}$$

The percentage of correct predictions was 95.3, and the Nagelkerke R^2 was 0.208.

To obtain the volumes of saw logs and pulpwood per trunk, the height of a tree of diameter $d_s, s = 1,\ldots,n$ is estimated from the data as follows:

$$h_s = \frac{39691d_s^2}{1000d^2 + 25683d + 37785} \tag{13}$$

Using the computation procedure described in Heinonen (1994), the sawn timber and pulpwood volumes per size class are given in Table 1. It is assumed that the maximum and minimum lengths of saw logs are 5.5 and 4.3 meters and that the minimum diameter for saw logs and pulp logs are 15 and 6 cm, respectively. The roadside price for saw logs equals 51.7 € m^{-3} and pulp logs 25 € m^{-3}. For harvesting cost we apply the empirically estimated models by Kuitto et al. (1994) (see Appendix A). These detailed models present cutting and hauling costs for thinnings and clearcuts under even-aged management and can be used to form a cost model for uneven-aged management as well. Costs depend on total harvested volume of saw logs and pulpwood and on the average volume of logs. The cost of artificial regeneration (including pre-commercial thinning) equals 1500 € per hectare.[1]

2.3. The optimization method

Applying the estimation results given in Eqs. (9)–(14), the problem of maximizing (6) subject to restrictions (1)–(5), (7a,b) and (8) constitutes a nonlinear programming problem. Further complications arise because the problem is potentially nonconvex and may yield multiple locally optimal solutions. The numerical analysis utilizes Knitro optimization software (Byrd et al., 1999, 2006; Wächter and Biegler, 2006) that applies gradient-based, state-of-the-art, interior point solution methods. About 200 initial guesses are chosen randomly to find the globally optimal solution. The infinite horizon solutions are approximated by applying finite time horizons. These are increased (up to 800 periods) until further lengthening the horizon does not change the solution trajectory toward a steady state or a stationary cycle. This trajectory toward the steady state or stationary cycle then represents an arbitrarily close approximation of the infinite horizon solution.

3. Results

3.1. Maximization of cubic meter yield

It is illustrative first to study the model properties in the simplest possible form, i.e. assuming that the aim is the maximization of physical yield. The highest physical yield is reached by applying the shortest possible cutting period, i.e. 3 years (one period). Based on this, Fig. 1 shows the development of four optimal solutions in basal area/cuttings space. The solutions approach a steady state with constant yield and basal area over time. These solutions are robust in the sense that varying the properties of the model, for example, increasing the effect of larger trees on the fraction of trees that move to the next size class, does not change the qualitative outcome (cf. Eq. (11)).

As shown in Fig. 1 and Table 2, second row (baseline solution), the optimal steady state yield equals 17.5 m^3 per 3 years; the maximized annual yield per hectare is about 6 m^3 The corresponding density measured in basal area before cuttings is about 128 m^3 and the basal area after cuttings about 109 m^2 (not shown). It should be noted that the basal area includes only trees with diameter at least 5 cm. According to the data, the basal area for trees smaller than 5 cm was typically about 1–2 m^2 implying that the total basal area per hectare varies between 12 and 15 m^2.

The development of the forest size class structure or the number of trees per size class before cutting is shown in Fig. 2. At the steady

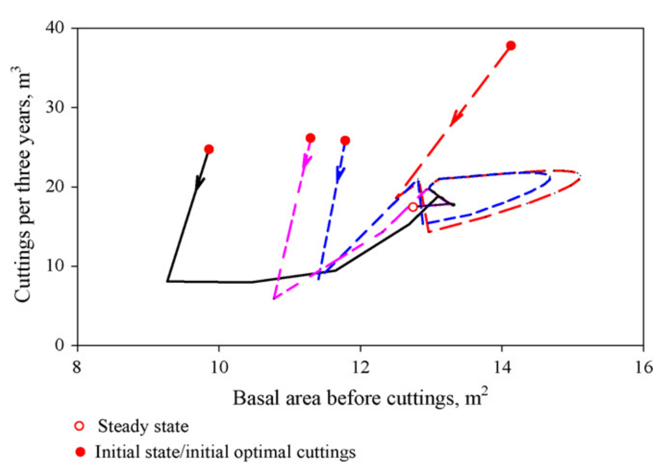

Fig. 1. Optimal development of basal area and cuttings toward the MSY steady state.

[1] T. Saksa, pers. communication, Finnish Statistical Yearbook of Forestry, 2006.

Table 2
The baseline solution and effects of stand ingrowth variation on density and cuttings.

	Ingrowth per 3 years (/ha)	Yield m^3/ha per 3 years	Basal area before cutting (m^2/ha)	Number of trees before cutting (/ha)	Cuttings per size class in number of trees (/ha)
Baseline solution	27	17.5	12.76	512	$h_7 \simeq 25$
Exogenous ingrowth: 10% increase to the baseline	30	18.58	15.80	661	$h_7 \simeq 25, h_8 \simeq 1$
Increased ingrowth: $x_{1,t+1} = 1.1(e^{2.1368+0.104\sqrt{N_t}-0.107y_t} - 1)$,	41.7	20.15	12.70	665	$h_6 \simeq 39$

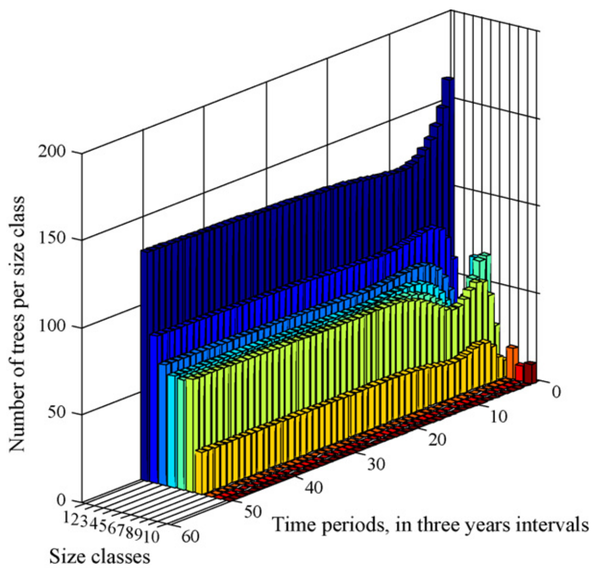

Fig. 2. Development of the size class distribution over time Number of trees before cuttings.

state it is optimal to cut trees when they reach a diameter class of 31 cm. The long-run size class structure deviates from the classical inverted J-form (Usher, 1966) since natural mortality for diameter classes 21–33 is almost zero. The ingrowth per 3 years is 27 trees and the number of trees harvested is 25, implying that about 7% of trees die before reaching the diameter class of 31 cm.

It would be interesting to understand the explanation for the somewhat low steady state stand density 12–15 m^3. It may be a consequence of either the negative effects of stand density (measured by basal area variables) on tree growth and survival or a consequence of low ingrowth. If low density is a consequence of the density effects on transition and mortality, a variation where ingrowth is given exogenously and is increased from its steady state level should shift cuttings to smaller trees and should not cause any major increase in basal area. The third row of Table 2 shows the optimal steady state levels of cuttings and other variables if ingrowth is given exogenously and increased by 10% from the baseline level. This variation increases the basal area and number of trees and switches cuttings toward larger size classes. The fourth row in Table 2 shows the outcome of increasing the endogenous ingrowth by 10%, i.e. multiplying the right hand side of Eq. (9) by a factor of 1.1. This variation causes a slight decrease in steady state basal area and a more than 10% increase in ingrowth and steady state cuttings. In addition, it becomes optimal to cut trees when they reach the diameter of 27 cm. Thus, in this case it is not optimal to let the basal area and stand density increase. In addition, decreasing the effects of basal area variables on growth and mortality in Eqs. (11) and (12), while keeping the ingrowth function intact, does not yield higher steady state basal area. Together this shows that in the optimal MSY solution stand density is kept at a low level because of its negative effect on ingrowth (Eq. (9)).

3.2. Comparing forest management systems in volume maximization

Fig. 3 compares the maximized (steady state) cubic meter yield from uneven-aged management and natural regeneration with the maximized yield from even-aged management based on artificial regeneration. The latter is computed by choosing the rotation period and harvesting strategy in order to maximize the average annual yield, i.e. by solving the problem:

$$\max_{\{T, h_{st}, s=1,\ldots,n, t=0,1,\ldots\}} V_2(\boldsymbol{x}_0) = \frac{\sum_{t=0}^{T}\sum_{s=1}^{n} h_{st}(\omega_{s1}+\omega_{s2})}{T}$$

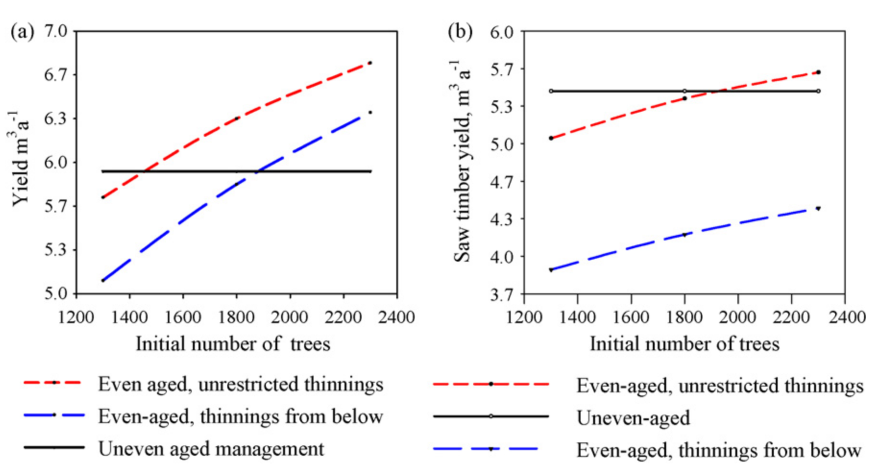

Fig. 3. (a and b). Comparison of maximum yields in even- and uneven-aged management: (a) maximization of the saw and pulp log volume and (b) maximization of saw log volume. Initial state: 24 years after regeneration 70% of trees enter size class 1 and 30% size class 2. Thinnings from below: thinnings allowed to four smallest size classes.

subject to restrictions (1)–(3) and (7a,b) where natural regeneration has been eliminated. In even-aged management the maximum yield depends on the number of seedlings. In what follows, the initial number of seedlings is varied between 1300 and 2300 (per ha). To specify the initial state it is assumed that after 26 years of clearcutting and 24 years of planting, 70% of trees have reach the smallest size class and 30% the next size class. For maximizing physical yield under even-aged management, it is optimal to apply thinnings from above. In this case the physical output from even-aged management exceeds the output from uneven-aged management given that the number of seedlings is higher than 1450 (see Fig. 3a). Given thinnings from below, i.e. if thinnings are restricted to the four smallest size classes only, even-aged management becomes inferior to uneven-aged management when the number of seedlings remains below 1900/ha.

Another possibility for such a comparison is to take the figures for maximized timber production from existing studies that apply models specified for even-aged management. In Hyytiäinen et al. (2005) the maximized yield for approximately similar forest type varies between 5 and 7.2 m^2 when the number of seedlings varies between 1300 and 2300 (per ha). This compares well with the figures obtained applying the model developed here.

Fig. 3b shows a similar comparison, if the aim is to maximize saw timber yield. In this case, even-aged management with unre-

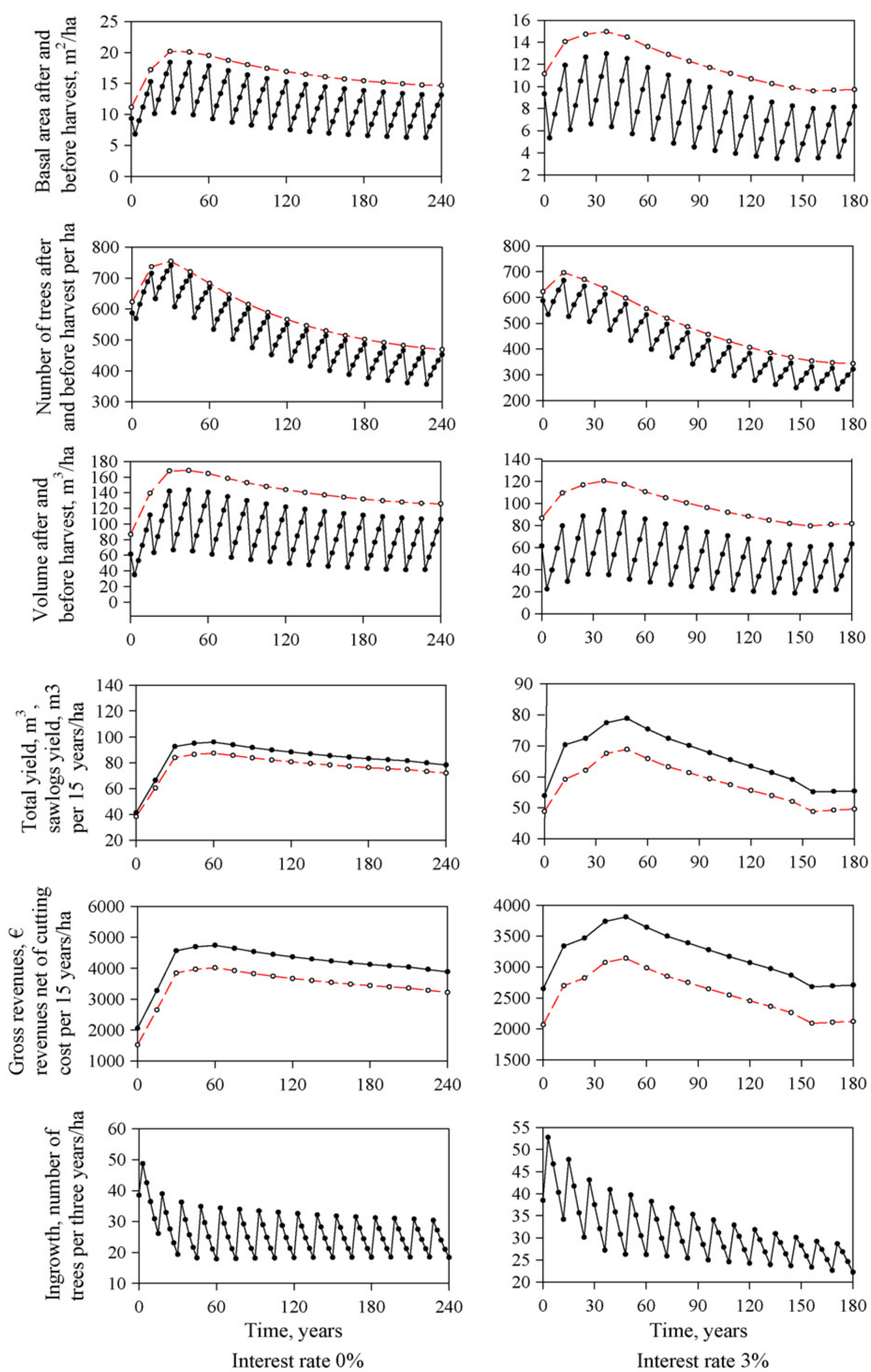

Fig. 4. Optimal uneven-aged solution with no discounting and 15-year cutting cycle and 3% interest rate and 12 years cutting cycle.

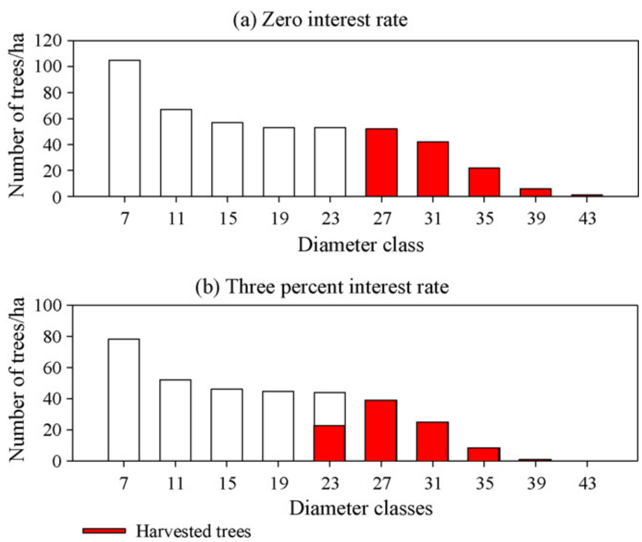

Fig. 5. (a and b) Optimal steady state size distribution and selection of harvested trees Cutting periods 15 years in Fig. 6a and 12 years in Fig. 6b.

stricted thinnings produces highest yield if the initial number of trees is above 1950/ha. If thinnings are restricted to the four smallest size classes, even-aged management yields lower output given any initial number of trees between 1300 and 2300 (per ha).

Along the uneven-aged steady state about 7% of the trees die. The percentage of dying trees under even-aged management increases with the initial number of trees and varies between 10% and 13%. Uneven-aged steady state cutting of 25 trees produces 17.5 m^3, i.e. yield per harvested tree equals about 0.7 m^3. The yield per harvested tree under even-aged management is 0.45, 0.38 or 0.33 m^3 given that the initial number of trees is 1300, 1800 or 2300 (per ha), respectively. Uneven-aged management yields higher yield per harvested tree because trees are harvested from the diameter class of 31 cm while under even-aged management it is optimal to harvest trees mainly from diameter class 27 cm. In addition, at the clearcut trees are harvested from all size classes. Such a harvest is not selective and thus decreases the cubic meter yield compared to the outcome of continuously accurate tree selection in uneven-aged management.

3.3. Optimal solutions under discounting and other economic factors

Fig. 4 shows the optimal solutions over time when the model includes harvesting costs, separate timber prices for saw logs and pulpwood and an interest rate equal to zero or 3%. The initial size distribution is $\boldsymbol{x}_0 = [282, 117, 73, 45, 28, 17, 11, 7, 4, 3]$. Given no discounting and the fixed harvesting cost of 300 €/ha it is optimal to apply a 15-year harvesting period, while the optimal cutting period with a 3% rate of interest is 12 years. Given 0% interest, the steady state average annual yield is 5.1 m^3 of which 4.8 m^3 is saw logs. Compared to the solutions that maximize volume output, the main reason in reduction in volume output is due to the 15-year harvesting interval. Fig. 5a and b shows the steady state size distribution of trees just before each cutting and optimal tree selection. Given no discounting each harvest removes all trees equal to or larger than 27 cm. When the interest rate equals 3% it is, in addition, optimal to cut about half from size class 24 cm. With the 3% interest rate, the annual total output and saw timber output are 4.6 and 4.1 m^3.

Given 0% interest, the total steady state revenue per cutting equals 3808 € and the revenue net of cutting cost 3147 €. This yields annual revenue per hectare equal to 210 €. Cutting and hauling costs per cubic meter become 8.6 €. The corresponding annual net income with a 3% interest rate is 178 € and cutting and hauling costs per cubic meter are about 11 €. With a 3% interest rate, the economic value of the forest per hectare just before and after cutting are 7143 and 5010 €. Because the stand density is lower with a 3% interest rate, the ingrowth level is somewhat higher compared with the optimal solution with 0% interest.

3.4. Comparing the economic superiority of silvicultural systems

The setup of this study enables various comparisons between the economic outcomes of even- and uneven-aged management. A basic set of comparisons is obtained by assuming some initial size class distribution and then computing whether it is optimal to choose even-aged management immediately or to switch to even-aged management after some optimized transition period or to continue with uneven-aged management forever.

Denote the value of bare land by V_{BL}. Thus the problem is to

$$\max_{\{h_{st},s=1,\ldots,n,t=0,\ldots,T,T\}} V_T(\boldsymbol{x}_0) = \sum_{t=0}^{T-1} b^t(R_t - C_t) + b^T(R_T - C_T^{cc} + V_{BL}) \tag{14}$$

subject to conditions (1)–(8) and where $b^T(R_T - C_T^{cc})$ is the net present value income from the first clearcut. If the optimal T in the solution of problem (14) is zero, it is optimal to apply even-aged management immediately. If the optimal T is above zero but finite, it pays to switch to even-aged management after some transitional period. If the optimal T is infinite, it is better to apply uneven-aged management forever and solving problem (14) yields the same present value for net revenues as solving the uneven-aged problem (1)–(8).

This setup can next be applied to computing a breakeven bare land value that implies equality between the even- and uneven-aged solutions. If this breakeven bare land value is higher than the actual bare land value, uneven-aged management is superior to even-aged management and *vice versa*.

The remaining question is how to obtain estimates for the value of bare land. One possibility is to compute the maximized bare land value by applying the model and data of this study. For this purpose, the objective function is specified as

$$\max_{\{T_F,h_{st},s=1,\ldots,n,t_i,i=0,1,\ldots,m\}} V_{BL}(\boldsymbol{x}_0) = \frac{\sum_{i=0}^{m} - w + b^{t_i}(R_{t_i} - C_{t_i}^{thin}) + b^{T_F}(R_{T_F} - C_{T_F}^{cc})}{1 - b^{T_F}}, \tag{15}$$

where w is regeneration cost, t_i, $i = 1, \ldots, m$ the dates for thinnings and T_F the Faustmann rotation period. Maximization of (15) is subject to the conditions (1)–(5), (7a,b) and (8) assuming, however, that trees are only regenerated artificially.

Instead of applying (15), another possibility is to use bare land values from existing studies. Hyytiäinen et al. (2010) perform detailed computations for optimal even-aged management for the Finnish Forest Extension Service Tapio to be used in developing official Finnish silvicultural guidelines. The computations for Norway spruce were based on the detailed single-tree model and optimization procedure developed in Valsta (1992) and Cao et al. (2005). The maximized bare land values of this study are given in Table 3, column (2) for interest rates between 1% and 5% (excluding the proportional profit tax of 29%). Pukkala (2005) applies a similar single-tree model and stumpage prices and presents the bare land values given in column (3). Hyytiäinen and Tahvonen (2002) used a simpler extended univariate model and presented bare land values (excluding proportional profit tax) shown in Table 3, column (4). The fifth column in Table 3 shows the bare land values obtained using the model at hand and the solution to (15). It is assumed that

Table 3
Bare land values for various interest rates according to different studies.

Study				
Interest rate	Hyytiäinen et al. (2010)[a]	Pukkala (2005)[b]	Hyytiäinen and Tahvonen (2002)[c]	This study[d]
0.5%	–		–	36979 €
1%	16197 €	12030 €	19202 €	15645 €
2%	4437 €	3393 €	5769 €	4867 €
3%	1396 €	1207 €	2105 €	1721 €
4%	116 €	514 €	652 €	324 €
5%	−290 €	–	–	−300 €

[a] Single-tree model; $p_1 = 53€,\ p_2 = 32€,\ w = 1000€$, number of seedlings 1800/ha.
[b] Single-tree model, stumpage prices, $p_1 = 40€,\ p_2 = 25€,\ w = 935€$, number of seedlings 1600/ha.
[c] Extended univariate model; $p_1 = 47€,\ p_2 = 32€,\ w = 500 - 1000€$, number of seedlings 1800/ha.
[d] $w = 1500$, initial state: x = 1750,0,. . .,0,(8 periods after the clearcut), thining interval 12–21 years.

26 years after the clearcut (24 years after the regeneration), size class 1 includes 1750 trees and that the cost of artificially planting 1 ha equals 1500 €. Separate harvesting costs for thinnings and clearcutting are given in Appendix A. Very similar harvesting cost functions are used in Hyytiäinen et al. (2010) and Hyytiäinen and Tahvonen (2002).

Figs. 6–8 compare these bare land values with the breakeven bare land value defined above. The figures are based on three different initial size class distributions. In Fig. 6 the initial state represents a steady state from the optimal uneven-aged solution (in the beginning of the period where cutting occurs) based on a 3% interest rate. The superiority of the even-aged management requires the bare land value to be above the solid black line (the breakeven bare land value). As shown only one bare land value estimate fulfills this requirement, i.e. the bare land value for the 1% interest rate from Hyytiäinen and Tahvonen (2002). All other bare land value estimates suggest that uneven-aged management is superior to even-aged management, i.e., the optimal T in (14) is infinite.

In Fig. 7, the initial size class distribution is $\boldsymbol{x}_0 = 450, 600, 450, 0, \ldots, 0$. This state may represent an even-aged stand about 35–45 years after regeneration. In this case, all bare land value estimates fall short of the breakeven curve implying that from this initial state it is optimal to apply uneven-aged management and rely on natural regeneration.

A third case represents an initial size class structure that may exist under even-aged management close to clearcut. In Fig. 8 most of the bare land value estimates exceed the breakeven bare land values given that the interest rate is below 2%. The reason why clearcutting and planting yield a higher present value of net revenues follows from the fact that this initial state is unfavorable for ingrowth and natural regeneration. Thus, given low interest rates, the costly regeneration is superior to (free but scanty) natural regeneration. However, the remaining question is whether, after

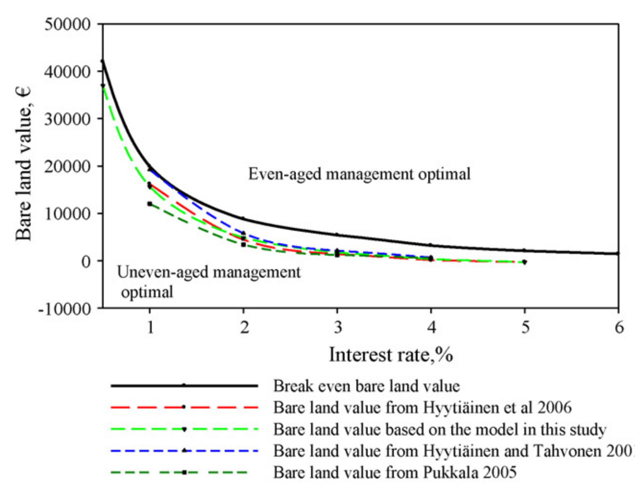

Fig. 7. Comparing the economic superiority of forest management forms. If bare land value is above the solid black line even-aged management yields higher present value of net revenues than uneven-aged management. Initial state: $\boldsymbol{x}_0 = 450, 600, 450, 0, 0, 0, 0, 0, 0, 0$.

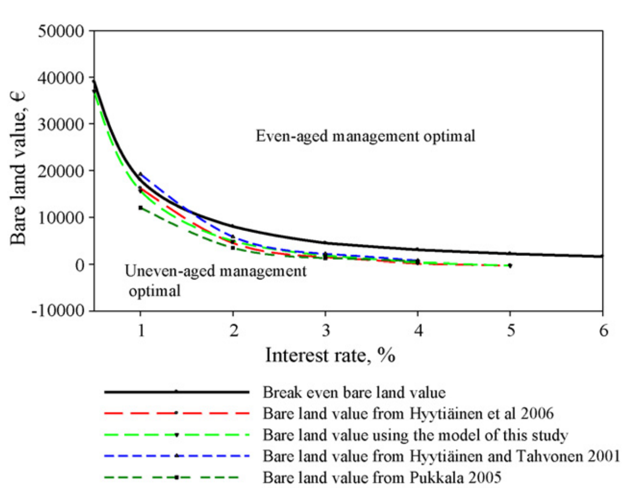

Fig. 6. Comparing the economic superiority of forest management forms. If bare land value is above the solid black line even-aged management yields higher present value of net revenues than uneven-aged management. Initial state: $\boldsymbol{x}_0 = 76, 53, 47, 45, 43, 35, 17, 3, 0, 0$.

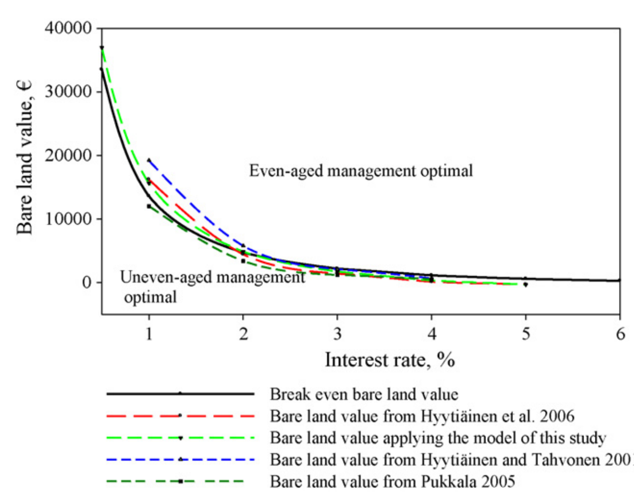

Fig. 8. Comparing the economic superiority of forest management forms. If bare land value is above the solid black line even-aged management yields higher present value of net revenues than uneven-aged management. Initial state: $\boldsymbol{x}_0 = 0, 0, 0, 0, 20, 80, 140, 135, 0, 0$.

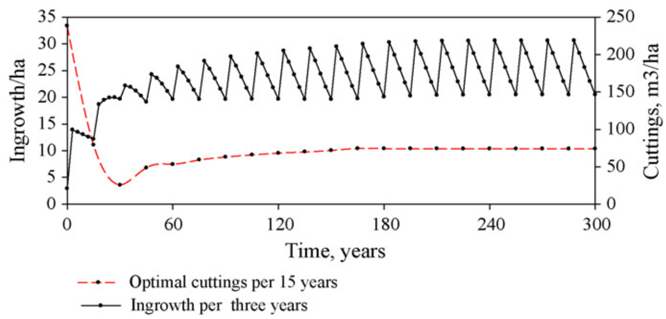

Fig. 9. Cuttings and ingrowth along the optimal solution. Initial state: $\boldsymbol{x}_0 =$ 0, 0, 0, 0, 20, 80, 140, 135, 0, 0; interest rate is 2%.

this clearcut, it is optimal to continue even-aged management forever. The answer appears to be negative since about 35–45 years after the regeneration the stand will reach a state like the one in Fig. 7. From this and similar states it is optimal to continue with uneven-aged management. Thus, given this initial state and interest rate below 2% the first best solution will slightly dominate the solutions shown in Fig. 8 and would consist an immediate clearcut followed by optimal uneven-aged management.

Assuming the same initial state as in Fig. 8, and 2% interest rate the optimal transition to uneven-aged management is shown in Figs. 9 and 10. Fig. 9 shows the development of total cuttings and ingrowth. As can be seen under uneven-aged management it is also optimal to apply rather heavy initial cutting. However, as shown in Fig. 10 the initial cutting is not a clearcut and it is optimal to maintain a fraction of large trees in the stand. The ingrowth in Fig. 9 is initially very low but it increases as the stand density is decreased by initial cuttings.

3.5. Effects of economic parameters on the optimal choice between forest management systems

According to an established result, in optimal rotation models without optimized thinnings the rotation period decreases with the rate of interest (Chang, 1984). In addition, given constant regeneration cost a high enough rate of interest implies that the bare land value becomes negative. This result does not change if the model includes thinnings from below. However, allowing unrestricted thinnings has somewhat surprising consequences. When the rate of interest is increased and the bare land value becomes negative it becomes non-optimal to cut the last tree because that will only lead to the realization of the negative bare land value. Thus the optimal rotation period becomes infinite. Obviously such an outcome would not occur if regeneration effort and cost could be chosen optimally. However, for Norway spruce there are very few (if any) models that could specify empirically estimated connection between the level of regeneration activities and stand growth. In spite of this problem, it is possible to apply the model from this study and analyze the effects of economic parameters on the superiority of the two management alternatives.

Fig. 11a (the lowest dotted line) shows optimal rotation when the interest rate is increased from 0% to 4% and thinnings are restricted to the six smallest size classes. Optimal rotation decreases with the interest rate confirming the established economic result. The dashed and solid curves show optimal rotation when thinnings are unrestricted and natural regeneration is included in the model. These changes increase the rotation length as expected. If the interest rate is zero, a planting cost equal to 1500 € is not too high and it is optimal to make such an investment instead of relaying only on natural regeneration. However, with the interest rate greater than or equal to 2.5%, the optimal rotation period becomes infinite, i.e. it becomes optimal to apply uneven-aged

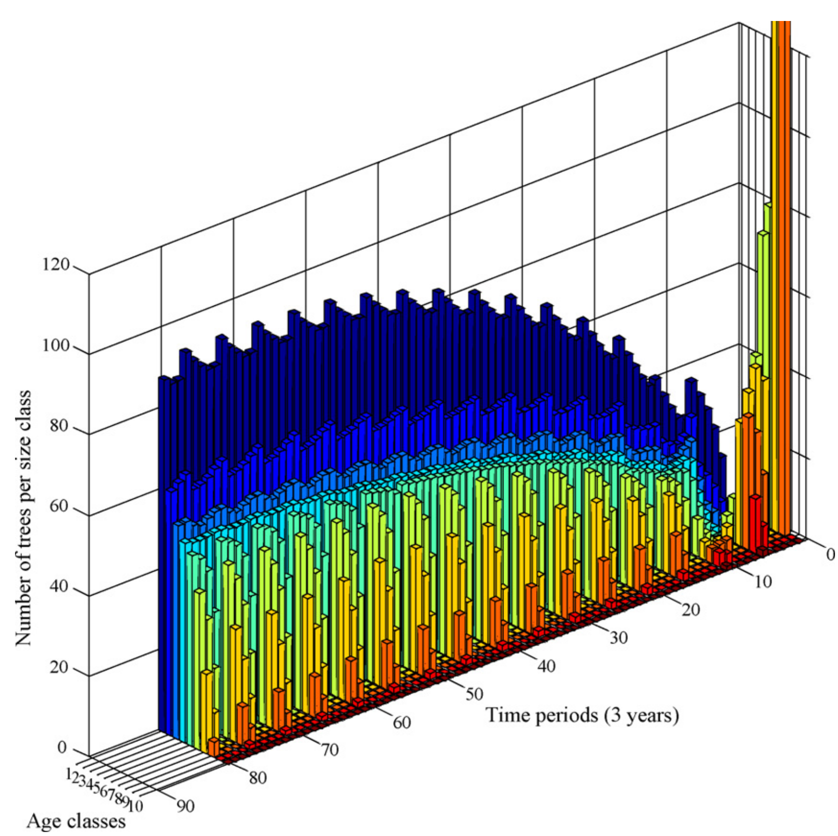

Fig. 10. Optimal development of tree size classes. Initial state: $\boldsymbol{x}_0 =$ 0, 0, 0, 0, 20, 80, 140, 135, 0, 0, $r = 2\%$.

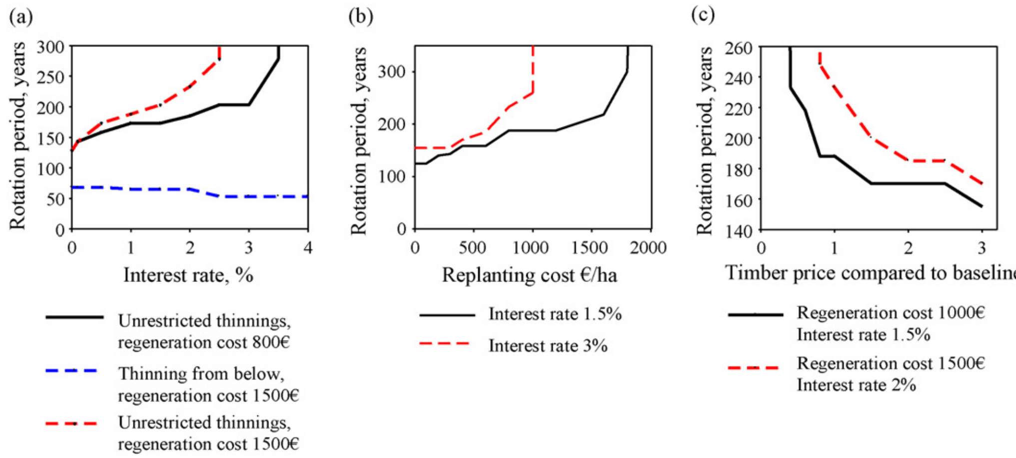

Fig. 11. (a–c) The effect of economic parameters on optimal rotation and choice between forest management systems. In (b) interest rate 1.5%. In (c) interest rate 1.5% and regeneration cost 1,000 €. In all cases $\boldsymbol{x}_0 = 300, 700, 600, 0, \ldots, 0$ and the delay between regeneration and initial state is 21 years.

forestry instead of even-aged forestry. With a lower regeneration cost (800 €), the rotation period is shorter and uneven-aged forestry becomes optimal when the interest rate exceeds 3.5%. The rotation period increases with the rate of interest because with higher interest it becomes optimal to apply costly regeneration investment less frequently. Fig. 11b shows the effect on increasing the regeneration cost and Fig. 11c the effect of timber prices. Increasing the regeneration cost or decreasing the timber price causes a switch from even-aged to uneven-aged management. These outcomes are in sharp contrast with typical even-aged economic models that do not include the trade off between artificial and natural regeneration or the possibility to continue thinnings from above without clearcuts.

4. Discussion and conclusions

Haight and Monserud (1990) studied the Northern Rocky Mountains mixed conifer stands. They found that in contrast to widely held views, both even-aged (with thinnings from above) and uneven-aged management are capable of yielding almost identical volume output. This study finds that in the case of Finnish Norway spruce, physical volume is maximized under even-aged management if the number of planted seedlings is above 1450. This result requires that even-aged management be applied with thinnings from above. However, the picture changes in favor of uneven-aged management if the goal is to maximize saw log volume. The superiority of even-aged management requires that the number of seedlings exceeds 1950. The interpretation behind this result is the finding that under uneven-aged management the average volume of harvested trees is almost twice as high as in even-aged management. A similar finding was reported by Andreassen and Øyen (2002). The superiority of thinnings from above to thinnings from below is in line with the findings by Lundqvist et al. (2007). In contrast to these earlier studies on Norway spruce, here the results are obtained applying empirical data together with a systematic optimization setup.

In Getz and Haight (1989), uneven-aged management becomes clearly superior when the aim is the maximization of present value stumpage revenues net of planting cost. They assumed that the interest rate equals 4% and that stumpage prices are independent of the forest management system, i.e. harvesting costs are neglected. The study at hand applies roadside prices and detailed harvesting cost specifications that take into account the cost differences between different kinds of harvests. Compared with clearcutting, harvesting costs in selective cuttings are higher but the optimization results show that this difference is decreased by the considerably higher volume of selectively harvested trees.

This study compares the economic superiority of the management alternatives by taking some initial state and then computing whether it is optimal to apply uneven-aged management forever or whether it is optimal to apply even-aged management immediately or after some future date. In general, applying uneven-aged management forever was found superior. The most critical factors behind its relative profitability are the levels of ingrowth, regeneration costs and interest rate. The superiority of uneven-aged management was independent on whether the profitability was computed using the same underlying model or whether it was taken from existing studies designed for the specific purposes of even-aged management.

The superiority of uneven-aged management was found to be sensitive to the initial state of the stand; an initially mature stand with unfavorable conditions for natural regeneration may be optimal to clearcut once and regenerate artificially. However, after this it is optimal to manage the stand toward uneven-aged structure, rely on natural regeneration and apply thinnings from above without clearcuts. This result is in line with the findings in Andreassen and Øyen (2002) who suggested that the conversion to uneven-aged management should be started earlier than in their experiments.

Given both volume maximization and economically optimal solutions the stand density remains at quite a low level (5–15 m^2). This was shown to be a consequence of the negative effects of density on ingrowth. Lundqvist et al. (2007) found an average ingrowth to the 5 cm class equal to 21 stems per year per hectare when the average basal area was about 11 m^2 This is considerably more than the ingrowth in the optimization runs of this study, where the annual ingrowth to the 7 cm class typically varies between 6 and 10 trees (with average basal area lower than 11 m^2). Even small increases in ingrowth imply considerable additions to yield and net revenues from uneven-aged management.

According to an established result, the optimal rotation period decreases with increases in interest rates (Chang, 1984). This study shows (numerically) that including thinnings and natural regeneration turns this result upside down: the rotation period increases (without bound) with increases in the interest rate. Intuitively it becomes optimal to make the costly regeneration investment less frequently. Similarly, increases in regeneration costs or decreases in timber prices do not only lengthen the rotation period but cause the optimal solution to switch from even-aged management to uneven-aged management. Similar results have not been shown previously in forest economic studies.

This study supports the earlier findings in Tahvonen (2007, 2009) and Pukkala et al. (2010) that challenge the general belief of unquestionable economic superiority of even-aged management for Norway spruce. Compared to Tahvonen (2007, 2009), this study shows that the relative profitability of the two management alternatives depend critically on ingrowth and compared to Pukkala et al. (2010) this study shows the importance of dynamic analysis and the initial stand state.

It becomes apparent that there is a trade off between costless natural regeneration and costly but perhaps more abundant artificial regeneration. This trade off depends crucially on discounting; higher interest rate favors uneven-aged management and vice versa. Another difference between the management alternatives follows from the fact that uneven-aged management leads to more accurate and economically more efficient tree selection compared to the rather crude clearcutting operation. The relative superiority of these systems is an outcome of a complex optimization problem that includes a great number of biological and economic details. Sharpening the understanding of these questions, calls for the inclusion of logging damage, a more reliable and detailed ingrowth model, non-timber values, and nonconstant cutting intervals into the models, to mention just some examples.

Appendix A.

Following Kuitto et al. (1994) and assuming logging costs per hour equal to 82.5 € and hauling cost per hour equal to 59.5 €, the costs for thinning and clearcutting in the case of even-aged management can be given as

$$C_t^{th} = 21.906306 + 3.3457762 H_t^{sawvol} + 25.5831144 + 3.77754938 H_t^{pulpvol} + \sum_{s=1}^{n} \left\{ 0.50001 + 0.59 vol_s - \frac{22.386}{vol_s 1000 + 85.621} \frac{h_{st}}{N_t} \right\} 2.1001366 N_t + 300,$$

$$C_t^{cc} = 26.350495 + 2.82183045 H_t^{sawvol} + 25.701440 + 3.33144 H_t^{pulpvol} + \sum_{s=1}^{n} \left\{ 0.44472 + 0.94 vol_s - \frac{146.17}{vol_s 1000 + 862.05} \frac{h_{st}}{N_t} \right\} 2.1001366 N_t + 300,$$

where H_t^{sawvol} and $H_t^{pulpvol}$ are the total volumes of sawlogs and pulpwood yields per cutting and vols is the total (commercial) volume of a stem from size class s.

The linear parts in both cost functions denote the hauling costs and the two nonlinear components the logging cost. In the case of uneven-aged management the cost function in (5) is formed by taking the hauling cost components from the thinning cost function and the logging costs using the logging cost component from the clearcut cost function multiplied by a factor equal to 1.15. This specification follows the suggestions in Surakka and Siren (2004). Fixed harvesting cost equals 300 €.

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